# ST JOSEPH'S UNIVERSITY BENGALURU-27



# **DEPARTMENT OF MATHEMATICS**

# SYLLABUS FOR POSTGRADUATE PROGRAMME

For Batch 2024-2026

# PART A

1	Title of the Academic Program	M.Sc Mathematics						
2	Program Code	(To be	(To be given by Examination Section)					
3	Name of the College	St. Jos	St. Joseph's College (Autonomous)					
4	Objective of the College	2.	<ol> <li>Academic Excellence</li> <li>Character Formation</li> <li>Social Concern</li> </ol>					
5	Vision of the College			a just, secular, democratic and economically sound society, which poor, the oppressed and the marginalized"				
6	Mission of the College	M1	be age	eph's College (Autonomous) seeks to form men and women who will ents of change, committed to the creation of a society that is just, r and democratic.				
		M2		ducation offered is oriented towards enabling students to strive for cademic and human excellence.				
		M3	enviro	college pursues academic excellence by providing a learning nment that constantly challenges the students and supports the ethical tof intellectual curiosity and ceaseless enquiry.				
		M4 Human excellence is promoted through courses and activities that students achieve personal integrity and conscientise them to the injure prevalent in society.						
7	Name of the Degree	Maste	Master of Science (M.Sc.) in Mathematics					
8	Name of the Department offering the program	Mathe	Mathematics					
9	Vision of the Department offering program	in lear	"The Department endeavors to be a center of excellence nurturing joyful curiosity in learning, enthusiastic creativity in research, passion to build a free, transparent and dynamic teaching learning community with a commitment to share and serve."					
10	Mission of the Department offering program	<ul> <li>Initiating students in the use of the power of abstraction.</li> <li>Enable students to perceive, enjoy and create patterns and the relationships that underlie the structures of Mathematics.</li> <li>Teach students to pose and solve meaningful Mathematical problems that delve into the service of humanity.</li> </ul>						
11	Duration of the Program	Two years (Four semesters)						
12	Total No. of Credits	Ninety	y Six					
13	Program Educational Objectives (PEOs)	PEO 1 The M.Sc programme is meticulously designed to impart essential knowledge in Mathematics with opportunities for specialization in all major areas of pure and applied mathematics as well as pursuing academic/industrial careers.						
		PEO 2 The non-academic outreach activities associated with the programme aim to inculcate in students a basic sense of responsibility and empathy towards social issues.						

		PEO 3	The teaching methodology (which focuses on the "why" rather than the "what"), adopted by faculty will instill a life-long learning attitude among students, to adopt new skills and techniques that will help overcome the problems that evolve with changing times.				
	]		The teaching pedagogy will encourage students to engage in an active learning process.				
14	Graduation Attributes		The Following graduate attributes reflect the particular quality and feature or characteristics of an individual, that are expected to be acquired by a graduate through studies at St. Joseph's College.  • Disciplinary knowledge • Critical thinking • Problem solving • Analytical reasoning • Research-related skills • Cooperation/Teamwork • Reflective thinking • Self-directed learning and Lifelong learner • Moral and ethical awareness/reasoning				
15	Program Outcomes (POs)	PO1	Students would have developed the ability to formulate and structure mathematical arguments through logical and deductive thinking.				
		PO2	Students would be competent in using programming languages like Python/ SageMath/R programming to solve problems related to the latest mathematical and industrial research.				
		PO3	Students would be able to work well as a team.				
		PO4	Students would be aware of their moral obligations to society and be willing to involve in tasks that work towards the greater good.				

# PART B

# M.Sc. Mathematics Curriculum

Courses and course completion requirements	No. of credits
Mathematics	96
Outreach activity and Ignitors	04

# **SUMMARY OF CREDITS**

	DEPARTMENT OF MATHEMATICS (PG) (2022-2024)							
Semester 1	Code Numb er	Title	No. of Hours of Instruc tions	Number of Hours of teaching per week	Numbe r of credits	Continuou s Internal Assessment (CIA/PIA) Marks	End Semes ter Marks	Total marks
Theory	MT 7121	Algebra I	60	04	04	50	50	100
Theory	MT 7221	Real Analysis	60	04	04	50	50	100
Theory	MT 7321	Linear Algebra	60	04	04	50	50	100
Theory	MT 7424	Ordinary Differential Equations	60	04	04	50	50	100
Theory	MT 7524	Discrete Mathematics and Graph Theory	60	04	04	50	50	100
Practica 1	MT 7P1	Linear Algebra and ODE with SageMath	44	04	02	25	25	50
Practica 1	MT 7P2	Graph Theory with SageMath	44	04	02	25	25	50
Tota	al Number	of credits:			24		•	
Semester 2	Code Numb er	Title	No. of Hours of	Number of teaching	Numbe r of credits	Continuou s Internal Assessment	End Semes ter	Total marks

			Instruc tions	hours /week		(CIA) Marks	Marks	
			tions					
Theory	MT 8121	Algebra II	60	04	04	50	50	100
Theory	MT 8221	Measure and Integration	60	04	04	50	50	100
Theory	MT 8321	Complex Analysis	60	04	04	50	50	100
Theory	MT 8421	Partial Differential Equations	60	04	04	50	50	100
Theory	MT 8521	Topology	60	04	04	50	50	100
Theory	MT 8624	Probability and Statistics	60	04	04	50	50	100
Practica 1	MT 8P1	Statistics with R Programming	44	04	02	25	25	50
Tota	al Number	of credits:			26			
Semester 3	Code Numb er	Title	No. of Hours of Instruc tions	Number of teaching hours /week	Numb er of credit s	Continuous Internal Assessment (CIA) Marks	End Seme ster Mar ks	Total marks
Theory	MT 9122	Functional Analysis	60	04	04	50	50	100
Theory	MT 9222	Classical and Continuum Mechanics	60	04	04	50	50	100
Theory	MTD E 9322 OR MTD E 9422	Graphs and Matrices OR Optimization Techniques	60	04	04	50	50	100
Theory	MT 9622	Mathematical Methods	60	04	04	50	50	100
Theory	MT 9722	Numerical Analysis	60	04	04	50	50	100

Practica 1	MT 9P1	Numerical Analysis with SageMath	44	04		02	25	25	50
Project	MT 9R1	Introduction to Mathematical Research	30	02		02	-	50	50
Tota	Total Number of credits: 24								
Semester 4	Code Numb er	Title	No. of Hours of Instruc tion	No. of teachi ng hours a week		umber of credits	Continuou s Internal Assessment (CIA) Marks	End Seme ster Mar ks	Total marks
Theory	MT 0122	Advanced Graph Theory	60	04		04	50	50	100
Theory	MT 0222	Fluid Mechanics	60	04		04	50	50	100
Theory	MT 0422	Finite Element Method	60	04		04	50	50	100
Theory	MTD E 0522 OR MTD E 0622	Differential Geometry OR Mathematical Modeling	60	04		04	50	50	100
Project	MT9R 2	Project	30	02		06	-	150	150
		IGNITORS/ OUTREACH							
Tota	al Number	of credits:				22	,		
			Total N	lo. of Cred	dits :	96			
	KEY WORDS: DE – Departmental Elective								

CORE COURSES (CC)					
Course Title	Code Number				
Algebra I	MT 7121				

Real Analysis	MT 7221
Linear Algebra	MT 7321
Ordinary Differential Equations	MT 7424
Discrete Mathematics and Graph Theory	MT 7524
Algebra II	MT 8121
Measure and Integration	MT 8221
Complex Analysis	MT 8321
Partial Differential Equations	MT 8421
Topology	MT 8521
Statistics	MT 8624
Functional Analysis	MT 9122
Classical and Continuum Mechanics	MT 9222
Mathematical Methods	MT 9622
Numerical Analysis	MT 9722
Advanced Graph Theory	MT 0122
Fluid Mechanics	MT 0222
Finite Element Method	MT 0422

DISCIPLINE SPECIFIC ELECTIVE COURSES (DSE)						
Course Title	Code Number					
Graphs and Matrices	MTDE 9322					
Optimization Techniques	MTDE 9422					
Differential Geometry	MTDE 0522					
Mathematical Modeling	MTDE 0622					

# SKILL ENHANCEMENT COURSE (SEC)

Course Title	Code Number
Linear Algebra and ODE with SageMath	MT 7P1
Graph Theory with SageMath	MT 7P2
Statistics with R programing	MT 8P1
Numerical Analysis with SageMath	MT 9P1
Introduction to Mathematical Research	MT 9R1

# Course Outcomes and Course Content Semester I

Semester	I
Paper Code	MT 7121
Paper Title	Algebra I
Number of teaching hours per week	04
Total number of teaching hours per semester	60
Number of credits	04

# **Objective of the Paper:**

To learn the concept of group action and use it to deduce important theorems in group theory regarding Class Equation, Automorphism, Inner Automorphism, Sylow's Theorem and other standard results. To understand the concept of irreducibility of polynomials. To understand the concepts of Euclidean Domain (ED), Principal Ideal Domain (PID) and Unique Factorization Domain (UFD).

# **Syllabus:**

## Unit 1

**A few examples of groups**: Dihedral Groups, Symmetric Groups, Matrix Groups, Quaternion Groups.

**Group actions**: Revisiting Cosets and Lagrange's theorem, Example showing that the converse of Lagrange's theorem is not true. Definition of group action and examples, Permutation representations. Cayley's Theorem and its generalization, A subgroup of index p where p is the smallest prime dividing the order of the group is normal, Group Acting on themselves by Conjugation, Class Equation, Conjugacy in Symmetric Group, A 5 is a simple group.

**(15 hours)** 

# Unit 2

**Automorphisms:** Automorphism group (Aut(G)), The quotient of a group by its center is isomorphic to a subgroup of Aut(G), Inner automorphism group, Computing Automorphism groups and Inner Automorphism groups, The automorphism group of finite cyclic group, Giving explicit descriptions of Automorphism groups.

**Sylow's Theorem:** Definition of p-subgroup, Sylow- p subgroup. Sylow's Theorem. Application of Sylow's Theorem: Groups of order pq, Groups of order 30, Groups of order 12, Groups of order p^2q, Groups of order 60. Simplicity of Alternating Group. Any simple group of order 60 is isomorphic to  $A_5$ .  $A_n$  is simple for  $n \ge 5$ . Sylow subgroups of  $D_{2n}$ ,  $S_n$ ,  $A_n$ ,  $SL_n((F_p))$ , and

problems from Exercise 4.5 (Page-146) from the main reference book that is Abstract Algebra by Dummit and Foote.

**External Direct Product:** Definition and examples of external direct products. Properties of external direct products.

**Fundamental Theorem of Abelian Group:** The fundamental theorem of finite abelian groups (without proof) and related problems. Greedy algorithm and related problems. Isomorphism classes of finite abelian groups. The fundamental theorem of finitely generated abelian groups (without proof).

(20 hours)

#### Unit 3:

**Polynomials Rings and Factorization of Polynomial Rings:** Polynomial ring, D[x] is an integral domain if D is an integral domain, Division algorithm. Remainder Theorem. Factor Theorem. Polynomials of degree n has at most n zeros counting multiplicity. Principal Ideal Domain (PID, If F is a field then F[x] is a PID.

Irreducible and Reducible Polynomial. Reducibility Test for Degree 2 and 3 polynomials, Gauss. Primitive polynomial. Content of a polynomial. Gauss lemma. Mod-p irreducibility Test, Eisenstein's criterion. Cyclotomic Polynomial. A Polynomial ring quotient an ideal generated by an irreducible polynomial is a field. Constructing fields with p^n elements, where p is a prime and n is an integer.

(13 hours)

## Unit 4:

# Euclidean Domains(ED), Principal Ideal Domains(PID) and Unique Factorization Domain(UFD):

Definition and Examples of ED, Non-Examples:  $Z[x], Z[\sqrt{-5}]$ , Concept of Greatest Common Divisor(GCD), Algorithm to find GCD of two elements in a ED, Definition and Examples of PID, Non-Examples:  $Z[x], Z[\sqrt{-5}]$ , GCD in a PID, Every nonzero prime ideal is a maximal ideal, If R[x] is a PID then R is a field, Definition of irreducible and prime elements, In an integral domain a prime element is always irreducible. In a PID an element is prime iff it is irreducible, Definition and Examples of UFD, GCD of two elements in a UFD.

Every ED is a PID, In a UFD a nonzero element is prime if and only if it is irreducible. Every PID is a UFD.

(12 hours)

# **TEXT BOOKS:**

- 1. D. S. Dummit and R. M. Foote. Abstract Algebra. Wiley. 2003.
- 2. J. A. Gallian. Contemporary Abstract Algebra. 4th Edition. Narosa Publishing. 2011.

# **REFERENCE BOOKS:**

- 1. C. S. Musili. Rings and Modules . 2nd Revised Edition. Narosa Publishing House. 1994.
- 2. I. N. Herstein. Topics in Algebra. 2nd Edition. Wiley. 1975.
- 3. I. S. Luthar and I. B. S. Passi. Algebra Volume-I Groups. Narosa Publishing House. 2013.
- 4. I. S. Luthar and I. B. S. Passi. Algebra Volume-II Rings. Narosa Publishing House. 2013.
- 5. J. B. Fraleigh. A first course in Abstract Algebra. 7th Edition Pearson Education India 2002.

- 6. M. Artin . Algebra. 2nd Edition Pearson Education India. 2017.
- 7. S. K. Mapa. Higher Algebra Abstract and Linear. Sarat Book House. 1972.
- 8. S. Lang. Algebra. 3rd Edition. Springer. 2002.

# **QUESTION PAPER PATTERN:**

Code number: MT 7121

Title of the paper: ALGEBRA I

**Paper Pattern:** Students will have to answer 5 out of 7 main questions. Each main question will be

worth a total of 10 marks.

The end semester question paper will have a weightage of 35% of the questions from the first half of the syllabus (the portions covered for the mid-semester examination) and a weightage of 65% of the questions from the second half of the syllabus (the portions not covered for the mid-semester examination).

CO1	Have developed good knowledge of Sylow's Theorems and irreducibility test for polynomials . Know the relation between ED, PID and UFD and different examples of them.
CO2	Understand the proofs of Sylow's Theorem, Fundamental Theorem of Abelian Group and the various irreducibility tests of polynomials.
CO3	Be able to apply Sylow's theorem to various problems in group theory, specially to check whether a group is simple and also to classify groups of certain orders. Be able to check the irreducibility of polynomials. Be able to check the nature (ED/PID/UFD) of a domain.
CO4	Be able to analyse which method of solution is the easiest to solve a given problem.
CO5	Be able to critique various proof methods for a particular theorem and explain why (or why not) one way is more useful than the other.
CO6	Be able to create examples and counter-examples particularly when working with the converse of certain theorems and implications.

Semester	I
Paper Code	MT 7221
Paper Title	Real Analysis
Number of teaching hours per week	04
Total number of teaching hours per semester	60
Number of credits	04

To comprehend the basics of Riemann Integration, improper integration and Sequences and Series of functions. To understand the fundamental concepts of metric spaces.

# **Syllabus:**

#### Unit 1:

**Countability**: Finite, Infinite, Denumerable, Countable, Uncountable sets. Examples, non-examples and properties of these sets. Countable union of countable sets is countable. The real line is uncountable. Cardinality of sets and related results. Cantor's Theorem.

(8 hours)

#### Unit 2:

**Riemann Integration:** Partition of a closed bounded interval. Upper and lower Darboux sums and Darboux integrals. Examples of integrable and non-integrable functions. Criteria for integration. Continuous and monotonic functions are integrable. Squeeze theorem. Riemann sums and Riemann definition of integral. Equivalence of the two definitions. Properties of Riemann integral. Lebesgue criterion. Indefinite integral. Fundamental theorems of calculus and mean value theorems. Integration by parts.

**(18 hours)** 

## Unit 3:

**Sequence and Series of Functions:** Pointwise convergence of sequence of functions. Different examples. Uniform convergence. Necessary sufficient conditions for uniform convergence. Uniform convergence and continuity. Uniform convergence and integration. Uniform convergence and differentiation. Weierstrass Approximation Theorem (without proof) and related problems. Power series. Radius of convergence.

(12 hours)

# Unit 4:

**Metric Spaces:** Notion of a metric space and examples. Open and closed sets in a metric space. Interior, exterior and boundary point. Limit and cluster point. Closure of sets. Bounded sets.

Distance between sets. Diameter of a set. Cantor's Intersection Theorem and its converse.

**Complete Metric spaces:** Sequences and subsequences in a metric space. Convergence of sequences in a metric space. Cauchy sequences in a metric space. Complete metric spaces. Subspaces of complete metric spaces. First and second category spaces. Baire's category theorem and its applications.

Continuous functions on metric spaces: Real valued continuous functions. Continuous functions between arbitrary metric spaces. Equivalent definitions of continuity. Examples of continuous functions. Uniform continuity.

**(22 hours)** 

#### **TEXT BOOKS:**

- 1. S. K. Mapa. Introduction to Real Analysis. 7th Edition. Sarat Book House. 2013.
- 2. D. R. Sherbert and G. Bartle. Introduction to Real Analysis. 4th Edition. Wiley. 2014.
- 3. D. Gopal, A. Deshmukh, A. S. Ranadive and S. Yadhav. An Introduction to Metric Spaces. 1st Edition. CRC Press. 2021.

## **REFERENCE BOOKS:**

- 1. W. Rudin. Principles of Mathematical Analysis. 3rd Edition. McGraw-Hill Education. 1976
- 2. J. M. Howie. Real Analysis. Springer India. 2001
- 3. S. K. Berberian. A first course in Real Analysis. Springer India. 1994
- 4. S. R. Ghorpade and B. V. Limaye. A course in Calculus and Real Analysis. 1st Edition. Springer. 2006
- 5. S. Shirali and H. L. Vasudeva. Metric Spaces. Springer. 2006
- 6. C. C. Pugh. Real Mathematical Analysis. 2nd Edition. UTM Springer. 2002
- 7. G. F. Simmons. Introduction to Topology and Modern Analysis. Tata McGraw-Hill Edition. 2004
- 8. S. Kumaresan. Topology of Metric Spaces. 2nd Edition. Narosa. 2005.
- 9. J. Munkres. Topology. 3rd Edition. PHI Learning Limited. 2012.

# **QUESTION PAPER PATTERN**

Code number: MT 7221

Title of the paper: REAL ANALYSIS

**Paper Pattern:** Students will have to answer 5 out of 7 main questions. Each main question will be worth a total of 10 marks.

The end semester question paper will have a weightage of 35% of the questions from the first half of the syllabus (the portions covered for the mid-semester examination) and a weightage of 65% of the questions from the second half of the syllabus (the portions not covered for the mid-semester examination).

# Course Outcomes: At the end of the Course, the Student should

CO1 Have good knowledge of the development of Riemann integration. Know the

	significance of uniform convergence over pointwise convergence of sequences/series of functions. Know examples and properties of finite, infinite, countable and uncountable sets as well as metric spaces.
CO2	Understand the geometry involved in the construction of the Riemann integral and in convergence. Understand the different techniques for checking if a given function is a metric and if a given set is countable/uncountable or open/closed.
CO3	Be able to apply the theorems learnt in each topic to solve problems.
CO4	Be able to analyze which method of solution is the easiest to solve a given problem.
CO5	Be able to critique various proof methods for a particular theorem and explain why (or why not) one way is more useful than the other.
CO6	Be able to create examples and counter-examples particularly when working with the converse of certain theorems and implications.

Semester	I
Paper Code	MT 7321
Paper Title	Linear Algebra
Number of teaching hours per week	04
Total number of teaching hours per semester	60
Number of credits	04

To study the general theory of vector spaces and linear transformations and understand different ways for representing linear transformations in simpler ways by means of matrices. To study vector spaces with some extra structures like inner product spaces, endomorphism rings, and some special type of linear transformations on some of these spaces like adjoint transformations, self-adjoint transformations, orthogonal transformations etc.

# **Syllabus:**

#### Unit 1:

**Vector spaces:** Review of solving system of equations and motivation for vector space structure on  $\mathbb{R}^n$ . Abstract Vector spaces – Definition and examples. Subspaces – Criterion for a subset to be a subspace, Examples, Union and Intersection of subspaces, Subspace generated by a set. Basis and dimension – Linear dependence and Independence, Definitions of a finite dimensional space, Basis and Dimension, Criterion for a subset to be a basis, Dimension of familiar subspaces. Linear transformations – Basic results, The rank-nullity theorem (Statement only), Algebra of linear transformations. Quotient spaces and the first Isomorphism Theorem (Statement only). Direct sum – Definition of internal direct sum and finding the dimension of direct sum of subspaces. Projection on a subspace along another subspace, The Idempotent operators. The matrix of a linear transformation.

**(15 hours)** 

#### Unit 2:

Canonical forms: Eigenvalues and eigenvectors – Definitions and basic results. The characteristic and minimal polynomials, Primary decomposition theorem, Annihilating polynomials, Cayley-Hamilton theorem, Computing minimal polynomials of some specific operators. Diagonalizable and Triangulable operators – Criteria for diagonalization and triangularebility. The Jordan form – The generalized eigenvectors and eigenspaces, The main theorem on Jordan canonical forms (Statement only), Problems on computing Jordan forms.

(20 hours)

#### Unit 3:

**Inner Product Spaces:** Inner products. Orthogonality. The Gram-Schmidt Orthogonalization Process. Orthogonal Complement. Adjoint of a linear operator. Self-adjoint and Normal operators. Unitary and Orthogonal matrices and Operator. Positive definite matrices. Test for positive definiteness. Polar and Singular value decomposition.

(20 hours)

## Unit 4:

**Bilinear forms and Quadratic forms:** Bilinear forms and Quadratic forms - Definitions and Examples. The matrix of a bilinear form and problems. (5 hours)

## **TEXT BOOKS:**

- 1. Vivek Sahai and Vikas Bist: Linear Algebra. 2<sup>nd</sup> Edition. Narosa Publishing House. 2013.
- 2. S. K. Mapa: Higher Algebra Abstract and Linear. revised 9th Edition. Sarath Book House. 2003.
- 3. A. J. Insel, L. E. Spence and S. Friedberg: Linear Algebra. 4th Edition. Pearson Education. 2003.
- 4. G. Strang: Linear Algebra and Its Application. 4th Edition. Cengage Learning. 2006.

## **REFERENCE BOOKS:**

- 1. A.R. Rao and P.Bhimasankaram: Linear Algebra. Hindustan Book Agency. Second Edition. 19 TRIM Series. 2010.
- 2. C. W. Curtis: Linear Algebra an Introductory Approach. 4th Edition. Springer. 1984.
- 3. D. C. Lay: Linear Algebra and its Application. 3rd Edition. Pearson Education India. 2009.
- 4. Seymour Lipschutz: Theory and Problems of Linear Algebra. SI (metric) edition. Schaum's outline series. McGraw Hill Publications. 1987.
- 5. K. Hoffman and R. Kunze: Linear Algebra. 2nd Edition. Prentice Hall India Ltd. 1978.
- 6. S. Lang: Linear Algebra. 3rd Edition. 11th Printing. Springer. 2004.
- 7. S. Kumaresan : Linear Algebra-A geometric approach. Prentice Hall India Private Limited. 2000.

# **QUESTION PAPER PATTERN**

Code number: MT 7321

Title of the paper: LINEAR ALGEBRA

**Paper Pattern:** Students will have to answer 5 out of 7 main questions. Each main question will be worth a total of 10 marks.

The end semester question paper will have a weightage of 35% of the questions from the first half of the syllabus (the portions covered for the mid-semester examination) and a weightage of 65% of the questions from the second half of the syllabus (the portions not covered for the mid-semester examination).

CO1	Have enhanced their knowledge of Linear Algebra by coming across a more general theory of Linear Algebra which is not restricted to finite dimensional vector spaces. Know more non-trivial examples of vector spaces and linear transformations and also know about vector spaces with some additional structures like inner product spaces, endomorphism rings of some fixed vector space etc.
CO2	Understand the crucial fact that to define a linear map, all one needs to do is to define any set theoretic map on a basis. Be able to understand the connection between algebra and geometry wherever possible.
CO3	Be able to apply the theorems learnt during the course for constructing algorithms for various computations.
CO4	Be able to analyze the given data and find ways for writing more efficient algorithms for computing the eigenvalues, for identifying the correct Jordan canonical form, for computing minimal polynomials etc.

Semester	I
Paper Code	MT 7424
Paper Title	Ordinary Differential Equations
Number of teaching hours per week	04
Total number of teaching hours per semester	60
Number of credits	04

To learn the basic techniques of solving ordinary differential equations and the stability of these solutions.

# **Syllabus:**

#### Unit 1:

Introduction. Fundamental Theorem (Basic existence theorem). Example of ODE without solution. First order linear differential equations. Linear dependence. Wronskian. Abel's formula. Fundamental sets of solutions. Equations with constant coefficients. Method of Undetermined Coefficients. Non-homogeneous equations. Growth and Decay Phenomena. Mixing Phenomena. Spring problem.

(15 hours)

#### Unit 2:

A Review of Power Series. Series solutions. Solution at an ordinary point. Analyticity of solutions at an ordinary point. Regular singular points. Solution at a regular singular point. The method of Frobenius. The gamma function. Bessel's Equation.

**(15 hours)** 

#### Unit 3:

Introduction to eigenvalue problems. The adjoint equation. Properties of self-adjoint problems. Sturm- Liovuelle's theorem. Characteristic Values and Characteristic Functions. Green's function. Self-Adjoint eigenvalue problems. Orthogonality of Characteristics function. Expansion of a function in a series of orthonormal functions. System of differential equations. Equilibrium points with example. First order systems. Systems with constant coefficients. Applications.

**(15 hours)** 

# Unit 4:

Nonlinear differential equations. First order differential equations. Exact solutions. Some special type of second order equations. Existence and uniqueness of solutions. The Phase Plane. Critical points. Stability for nonlinear systems (Liapunov). Perturbed linear systems.

**(15 hours)** 

## **TEXT BOOKS:**

- 1. S.L. Ross: Differential equations. John Wiley and Sons. New York. 3rd edition. 1984.
- 2. A. L. Rabenstein. An Introduction to Differential Equations. Academic Press. International Edition. 2014.

## **REFERENCE BOOKS:**

- 1. A.C.King. J.Billingham and S.R.Otto. Differential equations. Cambridge University Press. 2006.
- 2. E.A. Coddington and N. Levinson. Theory of ordinary differential equations. McGraw Hill. 1955
- 3. E.D. Rainville and P.E. Bedient. Elementary Differential Equations. McGraw Hill. New York 1969
- 4. G.F. Simmons. Differential Equations. Tata McGraw Hill Edition. New Delhi. 1974.
- 5. M.S.P. Eastham: Theory of ordinary differential equations. Van Nostrand. London. 1970.
- 6. S. J. Farlow. An Introduction to Differential Equations and their Applications. Dover Publications Inc. 2006.

# **QUESTION PAPER PATTERN**

Code number: MT 7424

Title of the paper: ORDINARY DIFFERENTIAL EQUATIONS

**Paper Pattern:** Students will have to answer 5 out of 7 main questions. Each main question will be worth a total of 10 marks.

The end semester question paper will have a weightage of 35% of the questions from the first half of the syllabus (the portions covered for the mid-semester examination) and a weightage of 65% of the questions from the second half of the syllabus (the portions not covered for the mid-semester examination).

CO1	Recognize and classify ordinary differential equations. Define Ordinary and Singular points of Differential Equations. Identify various methods to solve different kinds of ODE.
CO2	Interpret the type of solution of an ODE (for eg. Power series solution.) Understand how to find other solutions of an ODE if one of the solutions is given. Classify the 2 nd Order differential equations based on their properties and orthogonality
CO3	Apply the techniques such as the power series method, Green's function method or Lyopunov method to solve problems.
CO4	Analyze solutions of third order Differential equations, zeroes of solutions, critical points and their stabilities, and interpret it on the Phase plane.

Semester	I
Paper Code	MT 7524
Paper Title	Discrete Mathematics and Graph Theory
Number of teaching hours per week	04
Total number of teaching hours per semester	60
Number of credits	04

# **Objectives:**

To develop Mathematical maturity (i.e; the ability to understand and create mathematical arguments). To get an insight into how to utilize graph theoretical methods to analyse various connectivity patterns, data mining, image segmentation, clustering, image capturing and networking in the field of study and research.

# **Syllabus:**

#### Unit 1:

**Discrete Mathematics:** Mathematical logic, Rules of inference, Recurrence relations, Modeling with recurrence relations with examples of Fibonacci numbers, Solving linear and non-linear recurrence relations, Generating Functions, Counting Problems and Generating Functions, Using Generating Functions to Solve Recurrence Relations Representing relations using matrices and digraphs, Closures of relations, Transitive closures, Warshall's Algorithm, Partial Orderings, Lexicographic Order, Hasse diagrams, Maximal and Minimal elements, Lattices. Latin Squares.

**(15 hours)** 

## Unit 2:

**Fundamentals of graphs:** Definition of graph, Applications of graphs, Finite and Infinite graphs, Incidence and degree, Isolated vertex, Pendent vertex and Null graph. Directed graph, Types of digraphs, Digraphs and Binary relations, Directed paths and connectedness, Euler digraphs. Isomorphism, Subgraphs, A puzzle with multicolored cubes, Walks, Paths and Circuits, Connected graphs, disconnected graphs and components, Euler graphs, Operations on graphs, Hamiltonian paths and circuits, The traveling salesman problem.

**(15 hours)** 

#### Unit 3:

**Trees, Cut sets and Matrix representation of graphs**: Trees, Some properties of trees, Pendent vertices in a tree, Distance and centers in a tree, Rooted and binary trees, On counting trees, Spanning trees, Fundamental Circuits, Finding all spanning trees of a graph, Spanning trees in a weighted graph. Cut sets, Some properties of a cut set, All cut sets in a graph, Fundamental circuits

and cut sets, Connectivity and separability. Incidence matrix, Circuit matrix, Fundamental circuit matrix, Cut set matrix, Path matrix, Adjacency matrix.

**(15 hours)** 

## Unit 4:

**Coloring and Domination:** Coloring, The Four color problem, Vertex coloring, Chromatic number, clique number, Edge coloring, Edge chromatic number, The five color theorem, Chromatic polynomial, Ramsey Theory.

**Domination concepts:** Open neighborhood, Closed neighborhood, Dominating sets in graphs, Minimum dominating sets, Domination number. Bounds of domination number in terms of size, order, degree, diameter. Minimal dominating sets, Total domination, Total domination number.

(15 hours)

## **TEXT BOOKS:**

- 1. K. Rosen. Discrete Mathematics and its Applications. WCB McGraw-Hill. 8th edition. 2011
- 2. J.H.Van Lint and R.M. Wilson. A course on combinatorics. Cambridge University Press. 2006.
- 3. N. Deo: Graph Theory: Prentice Hall of India Pvt. Ltd. New Delhi, 1990.
- 4. G. Chartrand and Ping Zhang: Introduction to Graph Theory. McGrawHill, International edition (2005)24.
- 5. D.B. West, Introduction to Graph Theory, Pearson Education Asia, 2nd Edition, 2002.

## **REFERENCE BOOKS:**

- 1. F. Harary: Graph Theory, Addison -Wesley, 1969.
- 2. Charatrand and L. Lesnaik-Foster: Graph and Digraphs, CRC Press (Third Edition), 2010.
- 3. T.W. Haynes, S.T. Hedetniemi and P. J. Slater: Fundamental of domination in graphs, Marcel Dekker. Inc. New York.1998.
- 4. J. Gross and J. Yellen: Graph Theory and its application, CRC Press LLC, Boca Raton, Florida, 2000.
- 5. Norman Biggs: Algebraic Graph Theory, Cambridge University Press (2nd Ed.)1996.
- 6. Godsil and Royle: Algebraic Graph Theory: Springer Verlag, 2002.
- 7. J.A.Bondy and V.S.R.Murthy: Graph Theory with Applications, Macmillan, London, (2004).
- 8. J.P. Tremblay and R.P. Manohar . Discrete Mathematical Structures with applications to computer science. McGraw Hill. 1975.
- 9. H.J. Ryser. Combinatorial mathematics. American Mathematical Soc., 1963.
- 10. R.A. Beeler. How to Count: An Introduction to Combinatorics and Its Applications. Springer, 2015.

# **QUESTION PAPER PATTERN**

Code number: MT 7524

Title of the paper: DISCRETE MATHEMATICS AND GRAPH THEORY

**Paper Pattern:** Students will have to answer 5 out of 7 main questions. Each main question will be worth a total of 10 marks.

The end semester question paper will have a weightage of 35% of the questions from the first half

of the syllabus (the portions covered for the mid-semester examination) and a weightage of 65% of the questions from the second half of the syllabus (the portions not covered for the mid-semester examination).

CO1	Have knowledge of discrete mathematics and graph theory techniques and ability to solve problems of computer science, networking and problems involved in different fields of study and research.
CO2	Understand problems of Engineering and physical sciences, express them in terms of graphs and use theoretical knowledge to get solutions.
CO3	Apply fundamental concepts, definitions, lemmas and theorems in the appropriate situations to establish results of their research work.
CO4	Analyse various practical problems of real life and use mathematical thinking to find the solution.
CO5	Evaluate and synthesize research articles which are published.
CO6	Create mathematical modeling for the purpose of simplified representation of reality, to mimic the relevant features of the system being analysed.

Semester	I
Paper Code	MT 7P1
Paper Title	Linear Algebra and ODE with SageMath
Number of teaching hours per week	04
Total number of teaching hours per semester	44
Number of credits	02

To use SageMath, a Python based free and open source computer algebra system (CAS) to explore concepts in Applied Linear Algebra.

# **Syllabus:**

**Session 1:** Introduction to SageMath. Special Matrix constructors (Circulant Matrices, Cauchy Matrix, Permutation Matrix, Vandermonde Matrices, Tridiagonal Matrices, Idempotent Matrices, Nilpotent Matrices, Permutation Matrices, Stochastic Matrices, Doubly Stochastic Matrix.). Matrices over Rings such as  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ , Finite Fields etc.

Session 2: Properties of special matrices.

**Session 3:** Eigenvalue and Eigenvector. Bounds of Eigenvalues. Gershgorin Disc Theorem, Interlacing Lemma, Courant-Fischer, Weyl and Cauchy Theorems.

**Session 4:** Linear Transformation, Visualization of Linear Transformation: Rotation, Reflection, Scaling as a Linear Transformation.

**Session 5:** Inner Product, Norm of Vectors, Gram-Schmidt Orthogonalization.

**Session 6:** Matrix Decompositions I: QR and Cholesky Factorization.

Session 7: Matrix Decompositions II: Diagonalization, Singular Value Decomposition,

**Session 8:** Applications of SVD: Polar decomposition and Moore-Penrose inverse.

**Session 9:** Linear Differential equations. Plotting analytic and numerical solutions to linear differential equations.

**Session 10:** Nonlinear Differential equations. Plotting the solution and critical points.

Semester	I
Paper Code	MT 7P2
Paper Title	Graph Theory with SageMath
Number of teaching hours per week	04
Total number of teaching hours per semester	44
Number of credits	02

# **Objectives:**

To use SageMath, a Python based free and open source computer algebra system (CAS) to explore concepts in Graph Theory.

# **Syllabus:**

**Session 1: Introduction to graphs in SageMath:** Visualization - Plotting special graphs such as cycles, path, complete graph, Petersen graph. Graph parameters - order, size, vertex degrees, connectedness. Checking whether a graph is Eulerian, regular, degrees.

**Session 2: Directed graphs:** Digraphs, Creating and plotting digraphs, underlying graph of a digraph, connected digraph, weakly and strongly connected digraph, incidence matrix of a digraph, incidence matrix is unimodular, the order of the digraph is equal to the sum of rank of its incidence matrix and its number of components.

**Session 3: Adjacency matrix:** Adjacency matrix of an undirected graph, computing number of edges, number of triangles and number of k-walks of a graph using its adjacency matrix.

**Session 4: Laplacian matrix**: Laplacian matrix of a graph, relation between Laplacian matrix, distance matrix and adjacency matrix, Laplacian matrix as the product of incidence matrix and its transpose, laplacian matrix is positive semidefinite, computing rank of Laplacian matrix, row sum and column sum of laplacian matrix is zero, cofactors of any two elements of the Laplacian matrix are same

**Session 5: Trees 1:** Creating and plotting trees, tree on n vertices has n-1 edges, a graph is a tree if and only if there exists a unique path between any two vertices, every tree has at least two leaves.

**Session 6: Trees 2:** Finding eccentricities, radius, diameter and the centers of a tree, a tree contains maximum two centers, if a tree has two centers then they are adjacent, computing length of a path for a tree using Laplacian matrix, distance matrix of a graph, distance matrix of a tree has one positive eigenvalue and n-1 negative eigenvalue, relation between eigenvalues of distance matrix and Laplacian matrix.

**Session 7: Graph connectivity 1:** Deletion of a vertex, deletion of a edge, cut vertex, cut edge, characterisation of cut vertices in graph, every edge in a tree is a bridge, adding an edge in a tree forms exactly one cycle, center of any tree remains unchanged after removal of leaves.

Session 8: Graph connectivity 2: Vertex connectivity, edge connectivity, relation between vertex

connectivity and edge connectivity, relation between number of spanning trees in a graph, the graph obtained by deleting and contracting an edge, Matrix-Tree theorem.

**Session 9: Graph products:** Cartesian product, lexicographic product, direct product, distance formula for cartesian product, distance formula for lexicographic product, distance formula for direct product.

**Session 10: Graph coloring:** Find the chromatic number and chromatic polynomial of a graph, and verify the recurrence relation of the chromatic polynomial for a given pair of non-adjacent vertices, Edge chromatic number.

# **Semester II**

Semester	II
Paper Code	MT 8121
Paper Title	Algebra-II
Number of teaching hours per week	04
Total number of teaching hours per semester	60
Number of credits	04

# **Objective of the Paper:**

The core objective of the paper is to learn the elegant "Fundamental Theorem of Galois Theory" and to prove that a general quintic doesn't possess any formula (which involves addition, subtraction, multiplication, division and taking nth roots) to find its roots.

# Syllabus:

#### Unit 1:

**Chapter 1: Basic Theory of Field Extensions:** Characteristic of a field, Degree of an extension, Any polynomial f over a field F has a root in an extension K of F. Field generated by elements. Explicit description of K (basis of K as a vector space and understanding K as adjoining a root of an irreducible factor of f to F.

**Chapter 2: Algebraic Extensions:** Definition of Algebraic element and minimal polynomial of an algebraic element. Important theorems related to algebraic elements and minimal polynomials. Understanding quadratic extension over a field of characteristic not equal to 2. An extension is finite iff it is generated by finitely many algebraic elements. The property of an extension being algebraic is transitive.

Chapter 3: Splitting Field and Algebraic Closure: Definition of splitting field of a polynomial over a field and existence of splitting field. Computing splitting fields of different polynomials over specified fields. Introducing cyclotomic fields as splitting fields of  $x^n - 1$ . Primitive roots of unity. Uniqueness of splitting field. Definition of algebraic closure of a field and algebraically closed field. Existence of an algebraically closed field (without proof).

**(20 hours)** 

#### Unit 2:

**Chapter 4: Separable Extensions:** Definition of separable polynomial over a field. Derivative of a polynomial and criterion for a polynomial to be separable. Examples of separable and inseparable polynomials. Understanding when an irreducible polynomial is separable. Existence and Uniqueness of finite fields.

Chapter 5: Cyclotomic Polynomials: Defining the nth Cyclotomic Polynomial and computing cyclotomic polynomials. Proving that the  $n^{th}$  cyclotomic polynomial is an irreducible monic polynomial of degree  $\varphi(n)$  with integer coefficients.

(10 hours)

## Unit 3:

**Chapter 6: Automorphism group of a field extension:** Computing the automorphism group of different extensions. Definition of Galois extension. Different examples and non-examples of Galois extension.

**Chapter 7: Fundamental Theorem of Galois Theory:** An extension is Galois if and only if it is the splitting field of a separable polynomial. Fundamental Theorem of Galois Theory. Applying the fundamental theorem to find all the intermediate fields of different extensions. Finite fields over its prime sub-field are Galois extensions.

(20 hours)

#### Unit 4:

**Chapter 8: Insolvability of quintics:** Cyclotomic Extensions, Abelian Extensions, Galois Group of Polynomials, Symmetric functions, Discriminant of a polynomial. Characterizing the Galois group of a polynomial through its discriminant. A general polynomial of degree more than 4 can not be solved by radicals.

**(10 hours)** 

#### **TEXT BOOKS:**

- 1. D. S. Dummit and R. M. Foote. Abstract Algebra. Wiley. 2003.
- 2. J. A. Gallian. Contemporary Abstract Algebra. . 4th Edition. Narosa Publishing. 2011.

# **REFERENCE BOOKS:**

- 1. I. N. Herstein. Topics in Algebra. 2nd Edition. Wiley. 1975.
- 2. I. S. Luthar and I. B. S. Passi. Algebra Volume-IV Field Theory. Narosa Publishing House. 2013
- 3. J. B. Fraleigh. A first course in Abstract Algebra. 7th Edition Pearson Education India. 2002.
- 4. M. Artin . Algebra. 2nd Edition Pearson Education India. 2017.
- 5. S. Lang. Algebra. 3rd Edition. Springer. 2002.

# **QUESTION PAPER PATTERN**

Code number: MT 8121

Title of the paper: ALGEBRA II

**Paper Pattern:** Students will have to answer 5 out of 7 main questions. Each main question will be worth a total of 10 marks.

The end semester question paper will have a weightage of 35% of the questions from the first half of the syllabus (the portions covered for the mid-semester examination) and a weightage of 65% of the questions from the second half of the syllabus (the portions not covered for the mid-semester examination).

CO1	Have developed good knowledge of Splitting fields, Separable extensions, Finite Fields, Automorphism groups of an extension.
CO2	Understand the Fundamental Theorem of Galois Theory and insolvability of quintics.
CO3	Be able to apply the theory to compute splitting fields of different polynomials and automorphism groups of different extensions. Be able to apply Fundamental Theorem of Galois Theory to compute intermediate fields of any extension and its properties.
CO4	Be able to analyze which method of solution is the easiest to solve a given problem.
CO5	Be able to critique various proof methods for a particular theorem and explain why (or why not) one way is more useful than the other.
CO6	Be able to create examples and counterexamples of field extensions and the automorphism group of them.

Semester	П
Paper Code	MT 8221
Paper Title	Measure and Integration
Number of teaching hours per week	04
Total number of teaching hours per semester	60
Number of credits	04

To comprehend the basics of sigma algebras, measurable spaces and measurable functions. To understand the failings of the Riemann integral and learn how the Lebesgue integral is a generalization of Riemann integration.

# **Syllabus:**

#### Unit 1:

**Lebesgue Measure:** Failure of the Riemann integral - sequences of integrable functions whose limit is not integrable, non-interchangeability of limit and integral sign, need of a general theory of integration, need of a generalization of the concept of area. Lebesgue outer measure on  $R^n$  and its properties. Sigma algebras. Measures on a sigma algebra. Properties of a measure and Borel Cantelli Lemma. Measurable sets. Lebesgue measure on  $R^n$ . Properties of Lebesgue measure. Lebesgue sigma algebra. Non-measurable sets. Cantor set and sets of measure zero. Borel sets. Borel sigma algebra.

**(15 hours)** 

## Unit 2:

**Measurable Functions:** Measurable functions. Examples of measurable functions. Properties of measurable functions. Characteristic functions and simple functions. Approximating measurable functions by simple functions. Littlewood's three principles (includes Egorov's and Lusin's Theorems).

**(15 hours)** 

# Unit 3:

**Integration Theory:** Lebesgue Integral of simple functions and its properties. Lebesgue integral of bounded measurable functions supported on sets of finite measure and its properties. Bounded convergence theorem. Lebesgue integral on [a,b] is the same as the Riemann integral. Lebesgue integral of non-negative measurable functions and its properties. Fatou's lemma and monotone convergence theorem. Lebesgue integral for any measurable function. Lebesgue (Dominated) convergence theorem. Invariance Properties of the integral. Fubini's Theorem (without proof) and

its consequences. Product measure.

**(15 hours)** 

## Unit 4:

**Differentiation:** Differentiation of Monotone functions. Vitali covering lemma. Functions of Bounded variation. Differentiability of an indefinite integral. Absolute continuity.

**Lp spaces:**  $L^1$  spaces and Riesz-Fischer Theorem in  $L^1$ .  $L^p$  spaces. Holder and Minkowski inequalities. Convergence and completeness. Bounded linear functionals. Ries-Fischer theorem in  $L^p$ . Riesz representation theorem and its consequences.

**(15 hours)** 

# **TEXT BOOKS:**

- 1. E.M Stein and R. Shakarchi, Real Analysis Measure Theory, Integration and Hilbert Spaces. New age international publishers. 2010.
- 2. S. Kesavan, Measure and Integration. Hindustan Book Agency. 2019.

# **REFERENCE BOOKS:**

- 1. P.R. Halmos, Measure Theory, East West Press, 1962.
- 2. W. Rudin, Real and Complex Analysis, McGraw Hill, 1966.
- 3. P. K. Jain, V. P. Gupta, P. Jain, Lebesgue Measure and Integration. Anshan Publications, 2nd edition. 2012.
- 4. M. M. Rao, Measure Theory and Integration. CRC press, 2nd edition. 2004.
- 5. F. Morgan, Geometric Measure Theory. Academic Press, 5th edition. 2016.
- 6. H.L. Royden, Real Analysis, Macmillan, 1963.

# **QUESTION PAPER PATTERN**

Code number: MT 8221

Title of the paper: MEASURE AND INTEGRATION

**Paper Pattern:** Students will have to answer 5 out of 7 main questions. Each main question will be worth a total of 10 marks.

The end semester question paper will have a weightage of 35% of the questions from the first half of the syllabus (the portions covered for the mid-semester examination) and a weightage of 65% of the questions from the second half of the syllabus (the portions not covered for the mid-semester examination).

CO1	Have developed good knowledge of the development of the Lebesgue integral. Know some of the failings of the Riemann theory and how Lebesgue theory compensates for them.
CO2	Understand the techniques involved in checking properties of measures, sigma algebras and measurable functions.
CO3	Be able to apply the theorems learnt in each topic to solve problems, particularly

	those involving the Littlewood's three principles.
CO4	Be able to analyse which method of solution is the easiest to solve a given problem.
CO5	Be able to critique various proof methods for a particular theorem and explain why (or why not) one way is more useful than the other.

Semester	II
Paper Code	MT 8321
Paper Title	Complex Analysis
Number of teaching hours per week	04
Total number of teaching hours per semester	60
Number of credits	04

To comprehend the basics of Analytic function, its properties and its behavior in various domains. To get an understanding of some standard results related to the analytic function define on the entire space and on a unit circle.

# **Syllabus:**

#### Unit 1:

Analytic functions and the C-R equations. The Exponential, Sine and Cosine complex functions. Properties of Line integral. The closed curve theorem for Entire function. Cauchy Integral formula. Taylor series expansion of Entire function. Liouville Theorem. Fundamental Theorem of Algebra. Zeros of Analytic function. Gauss-Lucas Theorem.

(12 Hours)

#### Unit 2:

Power series representation for Analytic function. Uniqueness theorem. Mean Value theorem. Maximum Modulus theorem. Minimum Modulus theorem. Critical points and Saddle points. Open Mapping theorem. Schwarz Lemma. Morera's theorem.

(14 Hours)

#### Unit 3:

The General Cauchy Closed Curve theorem. Classification of Isolated singularities. Riemann Principle of Removable singularities. Casorati-Weierstrass Theorem. Laurent Expansion. Winding Numbers and Cauchy's Residue Theorem. Application of the Residue theorem. Argument principle. Rouche's Theorem. Hurwitz's Theorem.

(18 Hours)

#### Unit 4:

Evaluation of Definite Integral by Contour Integral Techniques. Application of Contour Integral Methods to Evaluation and Estimation of sums. Meromorphic functions. Conformal equivalence. Conformal Mapping. Riemann Mapping theorem. Harmonic functions. Mean-Value theorem for Harmonic Functions.

(16 Hours)

## **TEXTBOOKS:**

- 1. J. Bak and D.J. Newman, Complex Analysis, Springer. 2010.
- 2. E. Stein and R. Shakarchi, Complex Analysis, New Age International Publishers, 2010.

#### **REFERENCE BOOKS:**

- 1. J. B. Conway. Functions of one complex variable. Narosa. 1987.
- 2. L.V. Ahlfors. Complex Analysis. McGraw Hill. 1986.
- 3. T. W. Gamelin. Complex Analysis. Springer-Verlag. 2006.
- 4. R. Nevanlinna. Analytic functions. Springer. 1970.
- 5. E. Hille. Analytic Theory. Volume I. Ginn. 1959.
- 6. M. J. Ablowitz, A. S. Fokas. Complex Variables: Introduction and Applications. Cambridge Texts in Applied Mathematics. 2003.
- 7. S. Ponnuswamy. Foundations of Complex Variables. Alpha Science. 2nd edition. 2011.

## **OUESTION PAPER PATTERN**

Code number: MT 8321

Title of the paper: COMPLEX ANALYSIS

**Paper Pattern:** Students will have to answer 5 out of 7 main questions. Each main question will be worth a total of 10 marks.

The end semester question paper will have a weightage of 35% of the questions from the first half of the syllabus (the portions covered for the mid-semester examination) and a weightage of 65% of the questions from the second half of the syllabus (the portions not covered for the mid-semester examination).

CO1	Have developed a good knowledge about Analytic functions, its properties and its behavior in various domains. Students have basic knowledge of some standard results relating to it.
CO2	Have understood geometrically the properties of the analytic functions. Students have understood the different techniques to check a given function is analytical. Students also have understood to methods to determine the zeros and residues of the analytic function
CO3	Students will be able to distinguish the methods to determine the zeros and residues of the analytic function and explain which methods are more suited
CO4	Students will be able to create different mappings between complex spaces.

Semester	II
Paper Code	MT 8421
Paper Title	Partial Differential Equations
Number of teaching hours per week	04
Total number of teaching hours per semester	60
Number of credits	04

To understand the basics of solving standard partial differential equations. Also, to learn various boundary value problems with their applications.

# **Syllabus:**

#### Unit 1:

**First Order Linear Partial Differential Equations:** Origin of partial differential equations. Lagrange's equation Pp+Qq=R and problems based on each type. Integral surface passing through a given curve. Surfaces orthogonal to a given curve. Geometrical description of solutions of Pp+Qq=R.

(5 hours)

# Unit 2:

# **Second Order Linear Partial Differential Equations:**

Linear, Non-Linear and Quasi linear PDEs. Superposition principle. Classification of second-order linear partial differential equations into hyperbolic, parabolic and elliptic PDEs. Reduction of 2nd order linear PDEs to canonical forms and their general solution. Solution of linear Homogeneous and Non-Homogeneous PDEs with constant coefficients, PDEs reducible to constant coefficients, variable coefficients. Monge's method of Integration with distinct intermediate integral.

(20 hours)

#### Unit 3:

**Heat, Wave and Laplace equations:** Solution of heat, wave and Laplace equations by the method of separation of variables and integral transforms. Cauchy Problem (D'Alembert's and Riemann Volterra method). Duhamel's principle for wave and heat equation. Dirichlet, Neumann and Mixed problems in a rectangle for Laplace equation. Solution of heat, wave and Laplace equations in cylindrical and spherical polar coordinates.

(25 hours)

#### Unit 4:

Green's Function: Method of Eigenfunctions of expansion and method of Green's function

(Integral representation of the solution) for Heat equation, Wave equation and Laplace equation.

(10 hours)

## **TEXT BOOKS:**

- 1. M. D. Raisinghania. Advanced differential equations. S.Chand. 19th edition. 2018.
- 2. K.S. Rao. Partial Differential Equations. PHI Learning Private limited. 3rd edition. 2013.

# **REFERENCE BOOKS:**

- 1. V. Sundarapandian. Ordinary and Partial differential equations. McGraw Hill. 2012.
- 2. I. N. Sneddon. Elements of Partial Differential Equations. McGraw Hill Book company Inc. 2006.
- 3. M. G. Smith. Introduction to the theory of partial differential equation. Van Nostrand. 1967.
- 4. F. Treves. Basic linear partial differential equations. Academic Press. 1975.
- 5. L. Debnath. Nonlinear PDEs for Scientists and Engineers. Birkhauser. Boston. 2007.
- 6. F. John. Partial differential equations. Springer. 1971.

# **QUESTION PAPER PATTERN**

Code number: MT 8421

Title of the paper: PARTIAL DIFFERENTIAL EQUATIONS

**Paper Pattern:** Students will have to answer 5 out of 7 main questions. Each main question will be worth a total of 10 marks.

The end semester question paper will have a weightage of 35% of the questions from the first half of the syllabus (the portions covered for the mid-semester examination) and a weightage of 65% of the questions from the second half of the syllabus (the portions not covered for the mid-semester examination).

CO1	Have developed knowledge of first and second order partial differential equations along with different methods employed to obtain the solution for the same. Solution of heat, wave and Laplace equation are found under various boundary conditions involved.
CO2	Be able to understand different methods that can be incorporated to solve the multivariable function subjected to various boundary conditions.
CO3	Be able to analyse qualitative properties involved in the solution of the problem and adopt a suitable method to find the solution.
CO4	Be able to evaluate the solution of the problem that involves multivariable functions. For instance, problems involving propagation of heat or sound, fluid flow, mass transfer, wave theory.
CO5	Be able to create equations that impose relations between various partial derivatives of a multivariable function.

Semester	П
Paper Code	MT 8521
Paper Title	Topology
Number of teaching hours per week	04
Total number of teaching hours per semester	60
Number of credits	04

To comprehend the basics of Riemann Integration, Sequences and Series of functions. To generalize the concept of distance in the real line and thus understand the notion of Metric Spaces.

# **Syllabus:**

#### Unit 1:

**Introduction to Topology:** Definition and examples of topological spaces. Basis for a topology. Product Topology (finite product only). Subspace Topology. Neighborhoods and Limit points. Closed Sets and Limit points. Closure, Interior and Boundary of a set. Hausdorff Space. (Excluding the concept of finer and coarser, order topology, box topology).

(12 hours)

# Unit 2:

**Continuous Functions:** Definition and examples of continuous function. Equivalent definitions of continuity. Homeomorphism and examples. Pasting lemma. Maps into Product Spaces. Metric topology. Sequence Lemma.

(12 hours)

# Unit 3:

Connectedness and Compactness: Definition and examples. Union of connected sets having a point in common is connected. Image of a connected space under a continuous map is connected. A cartesian product of connected space is connected. Path connected spaces. Example of a topological space which is connected but not path connected (topologist's sine curve). Components and path components. (Excluding the concept of local connectedness).

Definition and Examples of Compact Spaces. Closed subspace of a compact space is compact. Compact subspace of a Hausdorff space is closed. Image of a compact set is compact under a continuous map. Tube lemma. The product of finitely many compact spaces is compact. Compactness and "finite intersection property". Compact subsets of the real line. Lebesgue number lemma. Uniform continuity and compactness. Limit point and sequential compactness. (Excluding the concept of local compactness)

(24 hours)

#### Unit 4:

Countability and Separation Axioms: First countable and Second Countable topological space. Hausdorff Space. Regular Space. Normal Space. Necessary and Sufficient condition for Regular and Normal Spaces. Subspace of regular is regular and subspace of normal is normal. Urysohn's Lemma. Urysohn Metrization theorem. Tietze Extension Theorem (without proof). Tychnoff Theorem (without proof).

(12 hours)

## **TEXT BOOKS:**

1. J. Munkres. Topology. Pearson Education India. 2nd Edition. 2007

# **REFERENCE BOOKS:**

- 1. J L. Kelley. General Topology. Van Nostrand. Princeton. 1955
- 2. J. B. Conway. A course in point set topology. UTM Series. Springer. 2013
- 3. K. D. Joshi. Topology. New Age International Private limited. 1983
- 4. M. A. Armstrong. Basic Topology. Springer India .1983.
- 5. G. F. Simmons. Introduction to Topology and Modern Analysis. Tata McGraw-Hill Education. 1963.

# **QUESTION PAPER PATTERN**

Code number: MT 8521

Title of the paper: TOPOLOGY

**Paper Pattern:** Students will have to answer 5 out of 7 main questions. Each main question will be worth a total of 10 marks.

The end semester question paper will have a weightage of 35% of the questions from the first half of the syllabus (the portions covered for the mid-semester examination) and a weightage of 65% of the questions from the second half of the syllabus (the portions not covered for the mid-semester examination).

CO1	Develop knowledge in basics, crucial definitions and theorems in topology and develop the ability to understand new definitions.	
CO2	Understand the basics of the field of topology, with emphasis on those aspects of the subject that are basic to higher mathematics.	
CO3	Be able to apply different proof writing techniques and write their own proofs	
CO4	Be able to analyze which technique is most useful in demonstrating a particular property of a topological space.	
CO5	Be able to critique various proof methods for a particular theorem and explain why (or why not) one way is more useful than the other.	
CO6	Be able to create examples and counter-examples particularly when working with the questions on homeomorphisms, connected and compact spaces.	

Semester	п
Paper Code	MT 8624
Paper Title	Probability and Statistics
Number of teaching hours per week	04
Total number of teaching hours per semester	60
Number of credits	04

This course aims to provide students with the foundations of probabilistic and statistical analysis mostly used in varied applications. It will also focus on the random variable, different types of distributions, sampling theory, to design a statistical hypothesis about the real world problem and to conduct appropriate test for drawing valid inference about the population characteristics.

# **Syllabus:**

## Unit 1:

Data Presentation: diagrammatic and graphical methods. Exploratory Data Analysis using descriptive measures and graphical tools. Univariate Analysis: Measures of central tendency, positional averages, measures of dispersion, moments, skewness and kurtosis – Definition, properties and problems related to it.

Probability theory: random experiment, simple events, sample space - types of events, probability of an event, rules of probability, conditional probability, Bayes' theorem.

**(15 hours)** 

#### Unit 2:

Probability distributions: random variables - discrete and continuous type, probability distribution table, Probability Mass Function and Probability Density Function Bernoulli, Binomial, Poisson, Geometric, Uniform, Exponential, Gamma and normal distributions - applications.

Joint distributions: Marginal and conditional distributions.

Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson and birth-and-death processes.

(15 hours)

# Unit 3:

Sampling methods - population and sample, parameter and statistic, concept of a random sample, simple random sampling, stratified sampling, systematic sampling, sample size determination.

Testing of hypothesis: null hypothesis, alternate hypothesis, test statistic, level of significance, p-value. Testing hypotheses about population mean, tests for proportions, tests concerning variance. Contingency tables, chi-square test for independence of attributes.

Correlation: Scatterplot, correlation coefficient and its properties, rank correlation, Test for correlation coefficient.

Regression: linear relationship, linear regression model, simple linear regression, Test for regression coefficients.

**(20 hours)** 

# Unit 4:

Analysis of variance: Completely randomized designs, randomized block designs and Latin-square designs.

(10 hours)

## **TEXT BOOKS:**

- 1. Vinay K. Rohatgi and A. K. MD. Ehsanes Saleh, "An Introduction to Probability and Statistics", Wiley-Inter-science Publication, John Wiley and Sons, Inc, New York, Second Edition, 2001.
- 2. Johnson, R.A., Miller, I and Freund J., "Miller and Freund's Probability and Statistics for Engineers", Pearson Education, Asia, 8th Edition, 2015.

#### **REFERENCE BOOKS:**

- 1. S. C. Gupta, V. K. Kapoor. Fundamentals of Mathematical Statistics. Sultan Chand and Sons. 1st Edition. 2020.
- 2. Walpole. R.E., Myers. R.H., Myers. S.L. and Ye. K., "Probability and Statistics for Engineers and Scientists", 9th Edition, Pearson Education, Asia, 2010.
- 3. Devore. J.L., "Probability and Statistics for Engineering and the Sciences", Cengage Learning, New Delhi, 8th Edition, 2014.

## **OUESTION PAPER PATTERN**

Code number: MT 8624

Title of the paper: PROBABILITY AND STATISTICS

**Paper Pattern:** Students will have to answer 5 out of 7 main questions. Each main question will be worth a total of 10 marks.

The end semester question paper will have a weightage of 35% of the questions from the first half of the syllabus (the portions covered for the mid-semester examination) and a weightage of 65% of the questions from the second half of the syllabus (the portions not covered for the mid-semester examination).

# Course Outcomes: At the end of the Course, the Student should

# **CO1**

Identify the types of data, use appropriate statistical terms to describe data and apply appropriate statistical methods to collect, organize, display, and analyze relevant data.

CO2	Define, explain the different statistical distributions, apply them to solve problems to find probabilities and predict the probability of certain events.	
CO3	Apply the concepts of hypothesis testing using appropriate test static and make valuable conclusions by proper evaluation.	
CO4	Analyze the different mathematical models with the help of statistical design and appropriate data, solve using appropriate methods and give inferences.	

Semester	II
Paper Code	MT 8P1
Paper Title	Probability and statistics with R Programming
Number of teaching hours per week	04
Total number of teaching hours per semester	44
Number of credits	02

To supplement the masters course with a basic knowledge of probability and statistics. To enable proficiency in R programming.

# **Syllabus:**

# Session 1: Basics of R programming and Diagrammatic Representation of Data:

Histogram, Frequency polygon, Dot Plots, Box Plot, Ternary Plot, Pie Chart, Bar Diagram.

# **Session 2: Univariate Analysis:**

Measures of central tendency, positional averages, measures of dispersion.

**Session 3:** Central Moments, skewness and kurtosis with diagrammatic representations.

## **Session 4: Probability Distributions I:**

Fitting of Data using Binomial, Poisson and Geometric distribution.

# **Session 5: Probability Distributions II:**

Fitting of Data using Uniform, Exponential and Normal Distribution.

## **Session 6: Markov Process**

Problems involving 1- step, 2-step Transition probabilities and Poisson birth and death processes.

# **Session 7: Parametric Testing of Hypothesis - I.**

Problems on Test Concerning Population Mean and Proportions for large samples.

# **Session 8: Parametric Testing of Hypothesis - II.**

Problems on Test Concerning Population Mean for small sample and test Concerning Variances.

## **Session 9: Bi-Variate Data Analysis:**

Problems on Computation of Correlation Coefficient (Karl Pearson's and Spearman's Coefficient of Correlation) and Simple Linear Regression.

# **Session 10: ANOVA**

Problems on Latin square design